One-channel EOG artifact correction: An analytic approach

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Abstract

This article approaches the problem of EOG artifact correction using one EOG channel from a biophysical point of view. It shows that recordings from one EOG channel are sufficient to correct artifacts from one-dimensional eye movements not exceeding 30°. We prove that the subtraction method "corrected EEG = measured EEG − backward propagation × measured EOG" yields the uncorrupted EEG trace up to scaling despite possible influences of forward propagation. Further, a special calibration paradigm (aligned artifact average, AAA) is investigated, and algorithms are presented to calculate the exact backward propagation. Experimental results from 13 subjects are shown, supporting the theoretical prediction of optimal correction.

Descriptors: EEG, EOG, artifact correction, regression, biophysical modelling

Artifacts from eye movements pose a serious problem in the analysis of the electroencephalogram (EEG). Conventionally, segments of EEG heavily contaminated by ocular artifact were rejected and excluded from further investigation (EOG rejection), and experiments were designed to avoid eye movement by telling participants to fixate on a certain point and try not to blink (fixation). Both methods have been shown to be problematic. For example, EOG rejection also removes EEG activity that correlates with high EOG activity (Verleger, Gasser, & Möcks, 1982) and adequate rejection criteria are hard to establish (Verleger, 1993). Further, fixation introduces a secondary task that has been shown to affect the CNV (Weerts & Lang, 1973), N1 (Verleger, 1991), and P300 (Ochoa & Polich, 2000; Verleger, 1991) components of the ERP.

Several techniques have thus been proposed in order to account for ocular artifact in the EEG, with no consensus as to the optimal technique(s). From the point of signal processing, two different classes of algorithms exist. The first group assumes that EOG signals can be measured and corrects each EEG channel by subtracting a channel-dependent portion of the EOG signal (Croft & Barry, 2000; Semlitsch, Anderer, Schuster, & Presslich, 1986). The second group utilizes algorithms based on either principal (Lins, Picton, Berg, & Scherg, 1993; Berg & Scherg, 1994) or independent (Gomez-Herrero et al., 2006; Makeig, Jung, Bell, Ghahremani, & Sejnowski, 1997) component analyses to decompose the recorded electrical activity into brain (EEG) and eye (EOG) activity. The corrected signal is then obtained by recomposing only the EEG part of the signal.

To date, there has not been a clear demonstration of superiority of either class of method, with theoretical advantages of the second group claimed by some (Jung et al., 2000), but no empirical data supporting this view and some suggesting the opposite (Wallstrom, Kass, Miller, Cohn, & Fox, 2004). One of the difficulties with reaching consensus on this issue is that it is difficult to determine whether a method has indeed accounted for the ocular artifact. Validation is difficult because we do not typically have situations where we know a priori what the uncontaminated EEG should look like, and thus a number of important (but often debated) assumptions need to be made. It is our view that in order to overcome this difficulty, effort needs to be directed toward improving both validation techniques and the theory underlying EOG correction. Some progress has been made recently with regard to improving validation (Croft, Chandler, Barry, Cooper, & Clarke, 2005) and simulation (Kierkels, van Boxtel, & Vogten, 2006) techniques, and it is the purpose of this article to provide a more rigorous theoretical treatment of EOG subtraction techniques.

The article is structured as follows: In the section “Biophysical Analysis of EOG Correction,” we review the EOG correction problem from a biophysical point of view and argue that EOG correction using the subtraction method is a theoretically sound approach. Our analysis is based on modeling the head as a linear volume conductor and the EEG and EOG signals as generated by dipoles (Berg & Scherg, 1991a, 1991b, 1994; Picton et al., 2000b). We briefly show that one-channel EOG correction is appropriate if eye movement is restricted to one plane and is small in size (<30°). We also turn to the problem of forward propagation and show that this does not affect the validity of the

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correction given certain assumptions. In the section “Calculation of the Backward Propagation Coefficient,” the aligned-artifact average (AAA) protocol is reviewed (Croft & Barry, 2000), where it is first conjectured that this specially designed calibration task satisfies the assumptions required for accurate propagation estimation (described in the section “Biophysical Analysis of EOG Correction”), and thus that it provides enough additional information to calculate the appropriate backward propagation coefficient $b$. It then describes four formulas that each exploit different aspects of the AAA and which will all converge to $b$ if the assumptions conjectured in the section “Calculation of the Backward Propagation Coefficient” about the AAA are accurate. In the section “Experimental Data” we then test empirically whether the assumptions of the section “Calculation of the backward propagation coefficient” are reasonable by determining whether $b$s from the four formulas do, in fact, converge, where it is shown that they do (and that the four formulas result in the same correction). Essentially, we conclude that our theoretical assumptions about the AAA also hold in practice, and this implies that we are able to solve the problem of optimal one-channel EOG correction by using the AAA experimental setup and any of the formulas given in the section “Calculation of the Backward Propagation Coefficient”. The article concludes with a practical discussion of the relative suitability of the four correction algorithms presented.

Biophysical Analysis of EOG Correction

In this section we analyze EOG artifacts from a biophysical point of view and discuss how the signals picked up at EEG and EOG electrodes are composed of electrooculographic and electroencephalographic parts. We show that if we were able to measure changes of vertical EOG activity directly, a single, time-invariant coefficient per electrode, the so-called backward propagation coefficient, would be enough to remove all EOG artifacts from the EEG data, but that this is not true for the EEG signal picked up at the EOG electrodes, because electric activation of the brain differs spatially over time, making a forward propagation up at the EOG electrodes, because electric activation of the brain acts on the EOG electrode $e$ at time $t$ is the superposition of the effects of the two dipoles, one for each eye:

$$E_{OEGe}(t) = \gamma(t) L(e,d_e) + \gamma(t) L(e,d_r).$$

Hereby, $\gamma(t)$ denotes the strength of the dipole at time $t$ and $L(e,d)$ encodes the lead field for dipole $d$ and electrode $e$ (i.e., how much of the field produced at $d$ is measured at $e$; Malmivuo & Plonsey, 1995), and the subscripts $l$ and $r$ denote the left and the right eye dipole, respectively. If we assume left and right eyes move equally, the activation pattern of the left and right eyes are the same: $\gamma(t) = \gamma(t) = \gamma(t)$. So Equation (1) becomes

$$E_{OEGe}(t) = \gamma(t) L(e,d) + L(e,d).$$

We define the absolute vertical backward propagation $B(e) = L(e,d) + L(e,d)$. The important point is that $B(e)$ is time invariant, but rather depends on the position of the electrode $e$ and the geometric and electromagnetic properties of the head. Note that, by the linearity of Equation (2), a change of reference or a (linear) combination of different EOG leads will alter the value of $B$, but not invalidate the argument.

Similar to Equation (1), we model the electroencephalographic activity generated by the brain as the different time courses of activation of a (possibly large) number of static dipoles. The activation of each dipole $d$ is mediated to the electrode $e$ by the lead field value $L(e,d)$. Thus, the total EEG activity picked up at $e$ is given by

$$E_{e}(t) = \sum_{i} \gamma_{i}(t) L(e,d_i).$$

Here, $\gamma_{i}(t)$ describes the time course of activation of dipole $i$. Now, by the linearity property of the brain as a volume conductor (Nayfeh & Brussel, 1985), the total potential at $e$ is given by the superposition of the eye potentials and the neuronal potentials:

$$E_{e}(t) = E_{OEGe}(t) + \gamma(t) B(e).$$

So to calculate the pure EEG signal at $e$, we need to subtract a fixed portion, namely, $B(e)$, of the eye activation from the total potential measurement at $e$. Note again that this portion, the absolute backward propagation coefficient, is time invariant.

Forward Propagation

Unfortunately, we have no direct access to the activation component $\gamma(t)$ of the eye, and we are left with methods such as attaching electrodes $\hat{e}$ above and below the eye and measuring the voltage difference between them. For these electrodes we then have the problem of forward propagation; besides electrooculographic activity, electroencephalographic activity is also picked up. In mathematical terms,

$$E_{e}(t) = E_{OEGe}(t) + \sum_{i} \gamma_{i}(t) L(\hat{e},d_i).$$

Matters simplify if all brain activity is synchronized, that is, all brain dipoles follow the same time course of activation. If this is the case, then, in mathematical terms, this amounts to $\gamma(t) = \gamma(t)$, for all $i$, and thus, Equation (5) becomes

$$E_{e}(t) = E_{OEGe}(t) + \gamma(t) \sum_{i} \sum_{i} L(\hat{e},d_i)$$

$$= E_{OEGe}(t) + \gamma(t) F(\hat{e})$$

and the coefficient $F(\hat{e})$, the absolute forward propagation, encodes the complete electromagnetic description of how the activation of the brain acts on the EOG electrode $\hat{e}$. Again, we like to remark that a change of reference will alter the value of $F$ but not invalidate our analysis.

In reality however, brain activity is seldom synchronized. So although it is justified to speak of a time invariant backward
propagation coefficient, the introduction of a fixed forward propagation coefficient is questionable. Nevertheless, for averaged event-related potentials, we can assume that contributions of unsynchronized dipoles cancel, and we are left with the activation from event-related dipoles only, which leaves a time invariant forward propagation coefficient $F$. Note that $F$ is still event specific; for different events different dipoles are active, with signals propagating differently to the EOG electrodes. Hence, we can only use the assumption of a time invariant forward propagation for the analysis of well-controlled ERPs, but not for EOG correction of general time courses.

**Perfect Correction**

Let us assume for a moment that the change of the EOG can be modeled with one static dipole per eye (as was discussed in the section “Backward Propagation”), and that we have a fixed forward propagation coefficient (as was discussed in the section “Forward Propagation”).

Let us denote by $MX$ the measured signal at an EEG electrode $e$, and by $MY$ the measured signal at an EEG electrode $e$. The true EOG signal at $e$, that is, the part of the signal picked up at the EOG channel stemming from the movement of the eyes, is denoted $X$, the true EOG signal at $e$, that is, the pure neuronal signal, by $Y$.

Now if we define the relative backward propagation coefficient $b$ as $B(e)/B(e)$, then the product $bX$ gives the EOG potential at the EEG electrode $e$. Similarly, if we define the relative forward propagation coefficient $f$ as $F(e)/F(e)$, then the EEG signal picked up at $e$ is $fY$. Thus, by (4) and (6) we can write

$$MX = X + fY$$

(7)

$$MY = Y + bX$$

(8)

The important observation here is that Equations (7) and (8) form a linear system that can be inverted if $f \cdot b \neq 1$:¹

$$Y = \frac{1}{1 - fb}(MY - bMX).$$

(9)

Hence, if we know the (relative) forward and backward propagation coefficients, we are able to accurately calculate the pure EEG signal at $e$ from the measured (contaminated) EEG signal and the measured (contaminated) EOG signal. We note at this point that a change in baseline for the measured signals ($MY$ is replaced by $MY + c_1$, $MX$ is replaced by $MY + c_2$) results in a change of baseline for $Y$ (namely, $Y$ changes to $Y + [(c_1 - b_c)/(1 - fb)]$). The shape of the correct signal $Y$ is thus not affected.

If we know only the forward propagation coefficient $b$ and assume a constant but unknown forward propagation $f$, we can use the formula

$$Y = MY - bMX$$

(10)

for EOG correction, as the error we make by using Equation (10) instead of Equation (9) is purely a scaling one. That is, all features of the true EEG trace are present and undistorted after using Equation (10) for correction, only the amplitude of the overall signal is changed. We note that the scaling error may not be constant for different EEG electrodes, leading to distorted scalp topographies. However, as we show in the next paragraph, changes in $f$ and $b$ from electrode to electrode have only a small effect on the scaling factor, limiting the distortion of scalp topographies.

If $f$ changes over time, because different brain regions become activated, the signal after correction using Equation (10) equals the true EEG signal times a time-varying scaling factor (namely $1/(1 - fb)$). If $f$ does not change too rapidly or if $|1 - fb| \approx 0$, the effects of the time variance of $f$ on the shape of the corrected EOG trace will be minor. For example, assume a large $b = 0.2$. If $f$ changes from 0.1 to 0.3, $1/(1 - fb)$ changes from 1.0204 to 1.0638; that is, the amplitude of the EEG signal changes by just 4% even though the forward propagation coefficient tripled. It should further be noted that by measuring the vertical EOG as the difference of voltage above and below the eye, a portion of the forward propagating neuronal signal cancels, reducing further the effects of forward propagation on the correction.

Hence, EOG correction using the subtraction method (10) is a theoretically sound practice even if given a changing and unknown forward propagation. The only information required is the backward propagation coefficient $b$, mediating between the measured signal at EEG electrode $e$ and the EOG contamination at EEG electrode $e$.

**Calculation of the Backward Propagation Coefficient**

The aim of this section is to give algorithms for the exact calculation of the (relative) backward propagation coefficient $b$. All algorithms are based on the AAA calibration paradigm, described first in the section “Calibration Paradigm,” where it is argued that this allows for accurate $b$ estimation. We then present four different methods to calculate $b$. We prove that all methods yield the exact $b$ given one critical assumption about the calibration paradigm. The adequacy of this assumption is then demonstrated in the section “Experimental Data.”

**Calibration Paradigm**

For the determination of the backward propagation, we employ a special experimental paradigm (aligned artifact average; Croft & Barry, 2000), which provides us with additional knowledge that helps disentangle forward and backward propagation behavior and can, in principle, be exploited to determine the backward propagation coefficient exactly.

The paradigm consists of a simple task, whereby a series (e.g., $N = 40$) of small magnitude (e.g., $10^7$) and duration (e.g., 800 ms) up and down eye movements are provoked. These data sets are then averaged to yield event-related potentials for up and down movements.

From the biophysical model it directly follows that—under optimal conditions—the activation of the vertical eye dipole for the up procedure is the negative activation from the down procedure. Hence

$$X_u = -X_d$$

(11)

Here, we have denoted the ERP of the up signal by a subscript $u$, the ERP of the down signal by a subscript $d$, and the equality is to be understood to hold for all time points.

Furthermore, we conjecture that the neuronal potentials of an up and a down movement are equal (or at least similar) (Croft et al., 2005); that is,

$$Y_u = Y_d$$

(12)

This assumption cannot be supported by simple biophysical models, because it includes ERP brain activity that is difficult to model. Therefore, as Equation (12) does not have the luxury of

¹As we will see in the section “Results,” $b$ ranges from $-0.1$ to 0.2, so this value represents the largest $b$ across all subjects and electrode sites for this study. If we assume a similar range for $f$, then $bf \leq 0.04 \ll 1$. 
biophysical support and as we need to demonstrate that both (11) and (12) are reasonable in terms of real experimental data, we need the four different algorithms for \( b \) presented in the following subsections and the experimental data from the section “Experimental Data” to present evidence for the adequacy of Equations (11) and (12).

However, for the remainder of this section we treat Equations (11) and (12) as true. This section also assumes that we can speak of a time-invariant forward propagation coefficient for the calibration task (Equation (7)), which was argued for in the section “Forward Propagation.”

Note that we do not assume any relationship between \( X \) and \( Y \), so it will be no problem if the saccade \( \{X, Y\} \) starts before the stimulus and its related neuronal potentials \( \{Y\} \). In the “Results” section we will see that this is indeed the case, because the participants often anticipate the stimulus.

**Minimization Approaches**

**Regression.** We will first analyze the conventional method of determining the backward propagation \( b \) by regression (Quilter, McGillivray, & Wadbrook, 1977). We show that under a relaxed version of assumptions (11) and (12), the regression algorithm closely approximates the correct \( b \). The calculations also provide insight into for which experimental conditions the regression algorithm fails to give the correct result.

For the analysis, we append the measured EOG and EEG traces for an up and a down movement: \( MX = \{MX_u, MX_d\}, MY = \{MY_u, MY_d\} \). The backward propagation coefficient is then estimated by linear regression:

\[
\sum_{t=1}^{N} (\alpha + \beta MX(t) - MY(t))^2 \rightarrow \text{min}. \quad (13)
\]

Here, \( N \) is the number of time points \( t \) of the aligned up–down procedure, \( \alpha \) the offset of the regression line, and \( \beta \) the estimate of the backward propagation coefficient \( b \). Note that \( \alpha \) is given as the mean of the \( \beta \)-corrected trace; that is, \( \alpha = \frac{1}{N} \sum_{t=1}^{N} (MY(t) - \beta MX(t)) \), so in fact Equation (13) finds the \( \beta \) for the corrected EEG trace with the minimal variance.

Solving the regression problem (13), we arrive at the following formula for \( \beta \):

\[
\beta = \frac{\text{cov}[MX, MY]}{\text{var}[MX]}. \quad (14)
\]

From Equations (7) and (8), Formula (14) can be rewritten to

\[
\beta = \frac{\text{cov}[X + fY, Y + bX]}{\text{var}[X + fY]} = \frac{(1 + bf) \text{cov}[X, Y] + b \text{var}[X] + f \text{var}[Y]}{\text{var}[X] + f^2 \text{var}[Y] + 2f \text{cov}[X, Y]} \quad (15)
\]

How does \( \beta \) relate to \( b \), the true backward propagation coefficient? A simple calculation yields that, given Assumptions (11) and (12), it holds that \( \text{cov}[X, Y] = \text{cov}[[X_u, X_d], [Y_u, Y_d]] = 0 \). So the first term in the nominator of (15) disappears and the denominator simplifies to \( \text{var}[X] + f^2 \text{var}[Y] \). If, furthermore, the forward propagation \( f \) is zero, then \( \beta = b \).

But also for \( f \neq 0 \), \( \beta \) approximates \( b \), because eye potentials \( X \) are usually much larger than neuronal potentials \( Y \), especially if we average them as per the AAA. Hence, \( \text{var}[X] \gg \text{var}[Y] \) and \( \text{var}[Y]/\text{var}[X] \approx 0 \), so

\[
\beta = \frac{b \text{var}[X] + f \text{var}[Y]}{\text{var}[X] + f^2 \text{var}[Y]} = \frac{b + \frac{\text{var}[Y]}{\text{var}[X]}}{1 + \frac{f^2 \text{var}[Y]}{\text{var}[X]}} \approx \frac{b}{1} = b.
\]

Note that we have also assumed \( f \ll b \) and \( f^2 \ll 1 \). For example, if \( b = 0.2, f = 0.3, \text{var}[X] = 1000, \) and \( \text{var}[Y] = 10 \), we calculate \( \beta = 0.203 \), which differs by only 0.003 from the correct value.

Thus, if we use AAA data for the calculation of \( b \) via linear regression and if our assumptions about the AAA paradigm hold, the result will closely approximate the true backward propagation coefficient.

**Amplitude minimization.** Even though historically the main reason for using the regression formula (13) was to find \( b \) by inspecting the similar shapes of the \( MX \) and \( MY \) traces, from the mathematical point of view we are solving a minimization problem. In this section we extend this minimization approach to obtain another approximative formula.

The idea is to use the fact that EOG artifacts are usually quite large compared to the EEG signal, and so we look for the \( \beta \) that minimizes the amplitude of the corrected signal most. In principle, this is similar to the original online potentiometers that were employed as a crude EOG correction method (Girton & Kamiya, 1973) and similar to a validation procedure of Croft et al. (2005). If we define

\[
h(\beta) := \sum_{t=1}^{N} (MY(t) - \beta MX(t))^2, \quad (16)
\]

where \( N \) is the number of time points, we find \( \beta \) by solving \( h(\beta) = 0 \), giving

\[
\beta = \frac{\text{E}[MXMY]}{\text{E}[MX^2]} = \frac{\sum_{t}^{N} MY(t) MX(t)}{\sum_{t}^{N} MX(t)^2}, \quad (17)
\]

where \( \text{E}[] \) is the expectation or mean.

How does \( \beta \) relate to \( b \)? Using Equations (7) and (8) we get

\[
\beta = \frac{\text{E}[(X + fY)(Y + bX)]}{\text{E}[(X + fY)^2]} = \left(\frac{\text{E}[XY](1 + bf) + b\text{E}[X^2] + f\text{E}[Y^2]}{2f\text{E}[XY] + \text{E}[X^2] + f^2\text{E}[Y^2]}\right), \quad (18)
\]

So if \( \text{E}[XY] = 0 \) and \( \text{E}[Y^2] \ll \text{E}[X^2] \), which is fulfilled for AAA data by a similar argument as in the section “Regression”, we get \( \beta = b \).

**Similarity Approaches**

Another property that we can exploit to generate other approaches is the conjectured similarity (12) of the corrected EEG signal \( Y \) for the up and the down movement of the eye. In essence, this is also the property exploited by Croft et al. (2005) in designing a EOG-trace independent validation method.

**Similarity of the mean.** As a consequence of assumption (12), the mean of the pure EEG trace for an up eye movement equals the mean of the pure EEG trace for the down eye movement.

\[2\text{Values rounded from corrected EEG sample data from the section “Experimental Data” for electrode Fpz, } f\text{-value estimated.} \]
We define

\[ h(\beta) := (E[Y_a(\beta, t)] - E[Y_a(\beta, t)])^2 \]
\[ = ((E[MY_a] - E[MY_d]) - \beta(E[MX_a] - E[MX_d]))^2. \]

(19)

We know that for \( \beta = h, h \) attains its minimum; that is, \( h(\beta) = 0 \). So we calculate \( \beta \) as the minimizer of \( h(\beta) \) by calculating the first derivative of \( h \), setting it to zero and solving for \( \beta \). This results in

\[ \beta = \frac{E[MY_a] - E[MY_d]}{E[MX_a] - E[MX_d]}. \]

(20)

By the definition of \( h(\beta) \) we know that if our strict assumptions (11) and (12) are true, then \( \beta = b \). But, in fact, all that is required for Equation (20) to yield the exact \( b \) is

\[ E[X_u] = -E[X_d], \]
\[ E[Y_u] = E[Y_d], \]

(21)

(22)

because then

\[ E[MY_u] - E[MY_d] = 2E[X_u], \]
\[ E[MY_a] - E[MY_d] = 2E[X_a]. \]

Thus, with Equation (20) we are able to calculate the exact \( b \) even if our assumptions about the AAA paradigm only hold in the weaker sense of first-order statistics.

**Similarity of the traces.** We can also make full use of the assumed equality \( Y_u(t) = Y_d(t) \) by forcing equality of the two corrected graphs, \( Y_u(\beta, t) \) and \( Y_d(\beta, t) \). So \( \beta \) is found by minimizing

\[ h_Y(\beta) := \sum_{t} \left( Y_u(\beta, t) - Y_d(\beta, t) \right)^2 \]
\[ = \sum_{t} \left( \frac{(MY_u(t) - MY_d(t))}{-D_Y(t)} - \beta \left( \frac{MX_u(t) - MX_d(t)}{-D_X(t)} \right) \right)^2 \]
\[ = N^2 E[(D_Y - \beta D_X)^2]. \]

(23)

We thus proceed as above by calculating the root of \( h_Y(\beta) \):

\[ h_Y(\beta) = 2N^2 E[D_X(D_Y - \beta D_X)], \]

(24)

so \( \beta \) is given by

\[ \beta = \frac{E[D_Y D_X]}{E[D_X^2]}. \]

(25)

We know of course that the strict assumption \( Y_u(t) = Y_d(t) \) and \( X_u(t) = X_d(t) \), for all \( t \), is unreasonable. But a short calculation shows that this is not necessary for \( \beta \) to equal \( b \). In fact, if we expand (25) using again (7) and (8), we arrive at

\[ \beta = \frac{(1 + hf)}{2fE[(X_u - X_d)^2]} + \frac{E[Y_a - Y_d]^2}{E[(X_u - X_d)^2] + f^2 E[(Y_a - Y_d)^2] + 1}. \]

(26)

We have already mentioned that the eye potentials of the up and the down procedures have opposite polarity. Furthermore, they are much larger than neuronal potentials, so \( \{E(Y_u - Y_d)^2\}/\{E[(X_u - X_d)^2]\} \approx 0 \). And similarly, the difference in magnitude is enough to ensure \( \{E(X_u - X_d)(Y_u - Y_d)\}/\{E[(X_u - X_d)^2]\} \approx 0 \). Hence, even if (12) is not strictly valid, the result of (25) should equal the true backward propagation coefficient when using the up-down paradigm.

**Experimental Data**

We have seen in the section “Biophysical Analysis of EOG Correction” that EOG correction using the subtraction method yields the correct result given that we are able to calculate the backward propagation coefficient \( b \). The section “Calculation of the Backward Propagation Coefficient” presented four different algorithms to calculate \( b \) and proved that they all give the correct result if data are obtained according to the AAA calibration paradigm and if assumptions (11) and (12) hold.

In this section we will now apply the four methods to data sets of 13 subjects and compare the correction across the methods using appended up-down data (AAA calibration), as well as with up and down data separately. All methods to calculate \( b \) have differing reliances on the AAA assumption. If each converges to give similar correction results empirically and to give diverging results if the assumptions are not met (without AAA), we are then able to conclude that the assumption was approximately met (with the AAA) and that the one-channel subtraction method is appropriate for accounting for vertical eye movement in the EEG.

**Methods**

Thirteen healthy subjects participated in the study (7 male, mean age 23.3 ± 2.5). Data from 68 channels were recorded and referenced to midway between Cz and CPz. Vertical EOG was defined as E1–E3, E1 and E3 being the electrodes above and below the left eye, respectively. Impedances for all sites were below 5 kΩ at the start of the recording. Gains of 2500 were used with a DC—200 Hz bandpass. Data were digitized at 1250 Hz.

Participants sat in an armchair and a computer monitor was placed 60 cm in front of them such that they looked perpendicularly at the center of the screen. A rectangular cursor appeared 100 times, alternating at top center and bottom center of the screen for 600 ms (SOA 800 ms). Participants were instructed to follow the cursor with the eyes.

Data were analyzed in MATLAB (The Mathworks, Inc.) using the EEGLAB toolbox (Delorme & Makeig, 2004) for preprocessing. Data was resampled at 250 Hz, re-referenced to the average of all scalp recordings, and low-pass filtered at 40 Hz. For each channel, average baseline over each up-down segment was removed and event-related potentials from 0.25 s before stimulus onset to 0.55 s after stimulus onset were calculated for the up and the down data separately.

First, we used averaged up-down data, according to the AAA paradigm, to calculate the backward propagation coefficients. We applied the four different algorithms (regression (13), minimal amplitude (17), similar expectations (19), and similar traces

\[ \text{The average reference was chosen because it best resembles a reference at infinity (Nunez & Srinivasan, 2006). Furthermore, compared with a single reference electrode, there are no scaling effects in the vicinity of the reference.} \]

\[ \text{Note that formulas (11) and (22) are valid for any time offset as long as the ERPs have 0.8 s length, because the up-down movement is performed cyclically with a period of 2 × 0.8 s.} \]
(25) to each channel of the recordings of each subject separately and calculated the corresponding backward propagation coefficient $b$.

Second, we applied the regression formula (13) and the minimum amplitude formula (17) once to the up data alone and once to the down data alone. (Note that the similarity algorithms from the section “Similarity Approaches” are specially designed for the up-down paradigm and hence cannot be applied meaningfully to the up or down procedure alone.) This allows us a comparison of how the algorithms behave if the AAA-assumptions are violated.

**Results**

**Correction using the up–down paradigm.** The mean and standard deviation of $b$ values from 13 subjects for 25 EEG electrodes and each of the four algorithms are presented in Table 1. We also calculated the range of the four algorithms for each electrode site, that is, the maximum $b$ minus the minimum $b$, and present the mean and standard deviation of this range over the 13 subjects. The average range over all 68 electrodes was calculated to be 0.0148 with a standard deviation of 0.0337. The average range over all 68 electrodes was calculated to be 0.0148 with a standard deviation of 0.0337. The average range over all 68 electrodes was calculated to be 0.0148 with a standard deviation of 0.0337.

We also used the different $b$s to correct the EEG traces using Formula (10). The resulting grand averages over 13 subjects for six selected electrode positions along the center line are shown in Figure 1. For comparison we also display the scaled average trace of the measured EOG. Note that the different amplitudes of the measured EEG require the use of different scales for the ordinate.

All corrected graphs show a high degree of similarity between the up and the down protocols. Furthermore, in all corrected graphs the amplitude is limited (5 μV), even though the contaminated original trace had an amplitude of up to 30 μV. The corrected traces show a clear evoked potential 210–230 ms after stimulus onset at Fz and Cz, with the peak amplitude for the eye-up movement occurring about 20 ms later than for the down movement. The peak amplitudes of the corrected traces differ for the up and the down traces by about 0.1 μV (Fz; sim mean, sim traces), 1 μV (Fz; regression, min amplitude), 0.6 μV (Cz; sim traces), and 2 μV (Cz; sim traces, regression, min amplitude). For Oz and Pz the traces show an ERP about 160 ms after stimulus onset with the same latency for down and up eye movements. The peak amplitudes of the corrected traces differ about 1 μV (Pz; sim mean), 2.25 μV (Pz; sim traces), 3 μV (Pz; regression, min amplitude), 0.8 μV (Oz; sim mean), and 0.2 μV (Oz; sim traces, regression, min amplitude).

**Separate correction of up and down experiment.** The results of a separate correction of the up and the down data are summarized in Table 2. As in the previous section, we took the mean from 13 subjects for each electrode and each algorithm for the up data and the down data. Furthermore, we computed the range of the two algorithms, applied to both data sets for each electrode site, and present the mean and standard deviation over 13 subjects. The mean range over all 68 electrodes and 13 subjects is 0.0613 with a standard deviation of 0.0899.

The results in Table 2 show that the $b$ values generally decrease from frontal to posterior regions, with a range across the EEG sites and the two algorithms of about 0.6. This value varies greatly between electrode sites. Furthermore, the difference between regression $b$s for the separate up and down data is generally larger than the difference between the minimal amplitude

<table>
<thead>
<tr>
<th>Electrode</th>
<th>Reg</th>
<th>Amp</th>
<th>Exp</th>
<th>Sim</th>
<th>Av Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fpz</td>
<td>0.178</td>
<td>0.179</td>
<td>0.169</td>
<td>0.175</td>
<td>0.175 ± 0.028</td>
</tr>
<tr>
<td>Fpz</td>
<td>0.187</td>
<td>0.187</td>
<td>0.173</td>
<td>0.183</td>
<td>0.183 ± 0.030</td>
</tr>
<tr>
<td>Fpz</td>
<td>0.171</td>
<td>0.172</td>
<td>0.161</td>
<td>0.167</td>
<td>0.168 ± 0.030</td>
</tr>
<tr>
<td>F7</td>
<td>0.052</td>
<td>0.053</td>
<td>0.049</td>
<td>0.052</td>
<td>0.052 ± 0.022</td>
</tr>
<tr>
<td>F3</td>
<td>0.067</td>
<td>0.067</td>
<td>0.069</td>
<td>0.068</td>
<td>0.068 ± 0.033</td>
</tr>
<tr>
<td>Fz</td>
<td>0.079</td>
<td>0.079</td>
<td>0.086</td>
<td>0.082</td>
<td>0.082 ± 0.026</td>
</tr>
<tr>
<td>F4</td>
<td>0.068</td>
<td>0.068</td>
<td>0.066</td>
<td>0.068</td>
<td>0.067 ± 0.039</td>
</tr>
<tr>
<td>F8</td>
<td>0.031</td>
<td>0.031</td>
<td>0.016</td>
<td>0.028</td>
<td>0.026 ± 0.038</td>
</tr>
<tr>
<td>FCz</td>
<td>0.054</td>
<td>0.055</td>
<td>0.071</td>
<td>0.060</td>
<td>0.060 ± 0.028</td>
</tr>
<tr>
<td>T7</td>
<td>–0.023</td>
<td>–0.023</td>
<td>–0.025</td>
<td>–0.023</td>
<td>–0.024 ± 0.030</td>
</tr>
<tr>
<td>C3</td>
<td>0.004</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003 ± 0.082</td>
</tr>
<tr>
<td>Cz</td>
<td>0.039</td>
<td>0.039</td>
<td>0.057</td>
<td>0.044</td>
<td>0.045 ± 0.024</td>
</tr>
<tr>
<td>C4</td>
<td>0.015</td>
<td>0.015</td>
<td>0.023</td>
<td>0.018</td>
<td>0.018 ± 0.012</td>
</tr>
<tr>
<td>T8</td>
<td>–0.030</td>
<td>–0.030</td>
<td>–0.030</td>
<td>–0.031</td>
<td>–0.030 ± 0.021</td>
</tr>
<tr>
<td>CPz</td>
<td>0.019</td>
<td>0.019</td>
<td>0.031</td>
<td>0.024</td>
<td>0.023 ± 0.014</td>
</tr>
<tr>
<td>P7</td>
<td>–0.045</td>
<td>–0.045</td>
<td>–0.035</td>
<td>–0.044</td>
<td>–0.042 ± 0.031</td>
</tr>
<tr>
<td>P3</td>
<td>–0.018</td>
<td>–0.018</td>
<td>–0.019</td>
<td>–0.015</td>
<td>–0.017 ± 0.034</td>
</tr>
<tr>
<td>Pz</td>
<td>–0.005</td>
<td>–0.005</td>
<td>0.004</td>
<td>–0.001</td>
<td>–0.002 ± 0.025</td>
</tr>
<tr>
<td>P4</td>
<td>–0.031</td>
<td>–0.031</td>
<td>–0.030</td>
<td>–0.029</td>
<td>0.030 ± 0.038</td>
</tr>
<tr>
<td>P8</td>
<td>–0.046</td>
<td>–0.046</td>
<td>–0.045</td>
<td>–0.046</td>
<td>–0.046 ± 0.034</td>
</tr>
<tr>
<td>POz</td>
<td>–0.024</td>
<td>–0.024</td>
<td>–0.014</td>
<td>–0.021</td>
<td>–0.021 ± 0.022</td>
</tr>
<tr>
<td>O1</td>
<td>–0.049</td>
<td>–0.049</td>
<td>–0.043</td>
<td>–0.048</td>
<td>–0.047 ± 0.029</td>
</tr>
<tr>
<td>Oz</td>
<td>–0.046</td>
<td>–0.046</td>
<td>–0.041</td>
<td>–0.045</td>
<td>–0.044 ± 0.033</td>
</tr>
<tr>
<td>O2</td>
<td>–0.050</td>
<td>–0.050</td>
<td>–0.046</td>
<td>–0.049</td>
<td>–0.048 ± 0.059</td>
</tr>
</tbody>
</table>

**Notes:** The first column gives the electrode name. Columns 2 to 5 give the backward propagation value using the regression algorithm (13), the minimal amplitude algorithm (17), and the similar tracking algorithm (25), respectively. All values are averaged over 13 subjects. We also calculated the average $b$ from the four algorithms. The means and standard deviations of this average $b$ over 13 subjects are displayed in column 6 (“Av”). Finally, we computed the range (max–min) of the four algorithms. The means and standard deviations of the range are displayed in column 7.
bs for the combined up–down data. Furthermore, the minimal amplitude bs generally have values in between the regression bs.

We also used the different bs to correct the EEG traces using Formula (10). The resulting grand averages for six selected electrodes along the center line are shown in Figure 2. For comparison we also display the scaled measured EOG and the correction using the regression formula applied to the appended up–down data.

In all corrected graphs the amplitude is limited to 5 μV, even though the contaminated original trace has an amplitude up to 30 μV. The evoked potentials described for the correction using the up–down paradigm are also discernible in the separately corrected traces. Compared to Figure 1, where we used the AAA paradigm to calculate b, there is less symmetry for the separately corrected traces for the up and the down experiments. The peak amplitudes for the P220 differ between the up and the down traces by about 1 μV (Fz; amp), 2.5 μV (Fz; regression), 2.5 μV (Cz; amp), and 4 μV (Cz; regression). The peak amplitudes for the P160 differ between the up and the down traces by about 3 μV (Pz; amp), 2.5 μV (Pz; regression), and 0.2 μV (Oz; amp, regression).

### Discussion

The present article has shown that the subtraction technique is a strong means of accounting for ocular artifact in the EEG, using the one-channel exemplar. It has shown, from a biophysical perspective, that the EEG signal at an electrode is contaminated by a fixed portion of the electrooculographic activity, and, mathematically, it has shown the following:

1. If we are able to measure the EOG and to determine this backward propagation coefficient b, a correction algorithm must have the form “corrected EEG = measured EEG − b * EOG.”

<table>
<thead>
<tr>
<th>Electrode</th>
<th>Reg up</th>
<th>Reg down</th>
<th>Amp up</th>
<th>Amp down</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fp1</td>
<td>0.194</td>
<td>0.146</td>
<td>0.186</td>
<td>0.166</td>
<td>0.058 ± 0.039</td>
</tr>
<tr>
<td>Fpz</td>
<td>0.206</td>
<td>0.163</td>
<td>0.192</td>
<td>0.176</td>
<td>0.060 ± 0.057</td>
</tr>
<tr>
<td>Fp2</td>
<td>0.187</td>
<td>0.163</td>
<td>0.178</td>
<td>0.161</td>
<td>0.053 ± 0.034</td>
</tr>
<tr>
<td>F7</td>
<td>0.059</td>
<td>0.053</td>
<td>0.054</td>
<td>0.050</td>
<td>0.027 ± 0.024</td>
</tr>
<tr>
<td>F3</td>
<td>0.070</td>
<td>0.058</td>
<td>0.067</td>
<td>0.068</td>
<td>0.053 ± 0.055</td>
</tr>
<tr>
<td>Fz</td>
<td>0.061</td>
<td>0.096</td>
<td>0.077</td>
<td>0.085</td>
<td>0.057 ± 0.053</td>
</tr>
<tr>
<td>F4</td>
<td>0.073</td>
<td>0.052</td>
<td>0.071</td>
<td>0.064</td>
<td>0.070 ± 0.080</td>
</tr>
<tr>
<td>F8</td>
<td>0.062</td>
<td>0.006</td>
<td>0.035</td>
<td>0.021</td>
<td>0.086 ± 0.111</td>
</tr>
<tr>
<td>FCz</td>
<td>0.026</td>
<td>0.079</td>
<td>0.050</td>
<td>0.067</td>
<td>0.074 ± 0.068</td>
</tr>
<tr>
<td>T7</td>
<td>–0.012</td>
<td>–0.021</td>
<td>–0.023</td>
<td>–0.024</td>
<td>0.037 ± 0.041</td>
</tr>
<tr>
<td>C3</td>
<td>–0.009</td>
<td>0.022</td>
<td>0.001</td>
<td>0.003</td>
<td>0.052 ± 0.043</td>
</tr>
<tr>
<td>Cz</td>
<td>0.014</td>
<td>0.059</td>
<td>0.035</td>
<td>0.051</td>
<td>0.066 ± 0.061</td>
</tr>
<tr>
<td>C4</td>
<td>0.013</td>
<td>0.020</td>
<td>0.014</td>
<td>0.021</td>
<td>0.032 ± 0.023</td>
</tr>
<tr>
<td>T8</td>
<td>–0.024</td>
<td>–0.058</td>
<td>–0.029</td>
<td>–0.032</td>
<td>0.059 ± 0.100</td>
</tr>
<tr>
<td>CPz</td>
<td>0.001</td>
<td>0.043</td>
<td>0.016</td>
<td>0.030</td>
<td>0.055 ± 0.045</td>
</tr>
<tr>
<td>P7</td>
<td>–0.045</td>
<td>–0.030</td>
<td>–0.050</td>
<td>–0.037</td>
<td>0.049 ± 0.047</td>
</tr>
<tr>
<td>P3</td>
<td>–0.028</td>
<td>0.017</td>
<td>–0.024</td>
<td>–0.010</td>
<td>0.053 ± 0.047</td>
</tr>
<tr>
<td>Pz</td>
<td>–0.014</td>
<td>0.019</td>
<td>–0.010</td>
<td>0.006</td>
<td>0.045 ± 0.031</td>
</tr>
<tr>
<td>P4</td>
<td>–0.020</td>
<td>–0.026</td>
<td>–0.033</td>
<td>–0.026</td>
<td>0.054 ± 0.077</td>
</tr>
<tr>
<td>P8</td>
<td>–0.037</td>
<td>–0.049</td>
<td>–0.047</td>
<td>–0.045</td>
<td>0.044 ± 0.063</td>
</tr>
<tr>
<td>POz</td>
<td>–0.037</td>
<td>–0.004</td>
<td>–0.028</td>
<td>–0.015</td>
<td>0.039 ± 0.038</td>
</tr>
<tr>
<td>O1</td>
<td>–0.051</td>
<td>–0.050</td>
<td>–0.050</td>
<td>–0.046</td>
<td>0.055 ± 0.068</td>
</tr>
<tr>
<td>Oz</td>
<td>–0.044</td>
<td>0.047</td>
<td>–0.047</td>
<td>–0.044</td>
<td>0.046 ± 0.054</td>
</tr>
<tr>
<td>O2</td>
<td>–0.037</td>
<td>–0.065</td>
<td>–0.049</td>
<td>–0.049</td>
<td>0.071 ± 0.127</td>
</tr>
</tbody>
</table>

**Notes:** The first column gives the electrode name. Columns 2 and 3 give the backward propagation value using the regression algorithm (13) applied to the up data and the down data, respectively. Columns 4 and 5 give the same for the minimal amplitude algorithm (17). All values are averaged over 13 subjects. Finally, we computed the range (max–min) of the two algorithms for the two data sets. The means and standard deviations of the range are displayed in column 6.

2. Forward propagation does not impair the quality of the correction as long as the correct b can be determined and the forward propagation does not change too rapidly.

3. If two assumptions about the AAA calibration hold, the regression formula (13) and the minimal amplitude formula (17) will closely approximate b, and the similar expectations...
formula (19) and the similar traces formula (25) will exactly calculate $b$.

The application of the four algorithms to data from 13 subjects revealed that the maximum difference between the calculated $b$s was on average 0.0148. As shown in Table 1, this value is fairly constant for all electrode sites and hence independent of the $b$ value (which ranged from 0.05 to 0.18 for the EEG electrodes). To put the range of the four algorithms in relation, we used the two approximating algorithms (regression and minimal amplitude) to calculate $b$ using only up data and only down data. Here, the range across the two algorithms and two data sets was more than four times as large, that is, 0.0613. We remark that this result is independent of the choice of baseline for the down and the up trace as the $b$ calculation via regression (Equation (13)) is not influenced by the baseline. Taking this as a measure, we conclude that the four algorithms from the section “Calculation of the Backward Propagation Coefficient” do indeed give similar results, provided we employ the AAA protocol. Although this is not a watertight proof, we conclude that our assumption about the similarity of the up and down EEG traces, Equations (11) and (12), are true.

This view is further supported from the traces of the corrected EEG, presented in Figure 1. We see that for all six depicted channels, the corrected EEG of the down movement (left half of each graph) approximately resembles the corrected EEG from the up movement (right half of each graph). The graphs show a clear evoked potential about 210 to 230 ms after the stimulus at Fz and Cz and an evoked potential 160 ms after stimulus at Oz and Pz. It was reported that there might be differences in neuronal signals of upward and downward saccades, especially in the posterior head regions (Picton et al., 2000a, 2000b). The graphs in Figure 1 confirm this finding, but also show that the differences are rather subtle. For example, the peak amplitude of the P220 ERP differs between the up and down traces by 1–3 μV, depending on the correction algorithm. As discussed in the section “Calculation of the Backward Propagation Coefficient”, all algorithms are well behaved if small deviations from the initial assumptions occur. Figure 1 shows also that corrected traces start and end well below the zero line. Because all our four algorithms show this behavior, we do not think that this is a sign of overcorrection. We rather take it to be an artifact from moving the eyes slightly sideways (not perfectly up and down) or caused by a change in the radial dipole strength (this change is small, because eye movement is limited, but nevertheless certainly present).

The (approximate) validity of assumption (12) implies that EOG correction using the here proposed subtraction method gives the optimal result when using the up–down paradigm and any of the formulas from the section “Calculation of the Backward Propagation Coefficient” to calculate the backward propagation coefficient.

Until now, EOG correction procedures have often been applied without sufficiently relating them to the underlying bioelectrical processes or investigating and verifying the assumptions for their convergence against the correct results. This resulted in a number of different correction approaches (e.g., subtraction methods, blind source separation, with/without calibration, frequency vs. time dependent methods, different number of EOG channels, different vs. similar correction coefficients for blink and saccade data). Although there were several attempts to compare them (Croft et al., 2005; He, Wilson, Russell, & Gerschutz, 2007; Kierkels et al., 2006), there has not been agreement as to which procedure performs best. We believe that this matter can only be settled by thorough investigation of the bioelectric processes underlying EOG artifacts and the convergence assumptions of the correction algorithms, as we have demonstrated in this article for EOG artifacts stemming from one-dimensional eye movement and correction using the subtraction method.
The discussion of EOG artifacts in the section “Biophysical Analysis of EOG Correction” has been limited to small eye movements in one direction, resulting in a correction formula employing only one EOG channel. If the eye rotates freely, all three components (vertical, horizontal, radial) of the eye dipole change. Hence, an EOG correction formula for general eye movements of large angular amplitude must address all three of them (Elbert, Lutzenberger, Rockstroh, & Birbaumer, 1985). The matter is further complicated by the fact that the three EOG components cannot be recorded independently; any arrangement of EOG electrodes will always pick up contributions from all components at all leads (to a certain degree). Addressing this scenario with the same rigor as we have done for the limited one-dimensional eye movement is possible, but it is beyond the scope of this article and will hence be published separately.

As the section “Calculation of the Backward Propagation Coefficient” presented three methods additional to the more standard least-squares regression method that differentially rely on certain assumptions of the \( b \) estimation process, the results from the section “Experimental Data” offer the opportunity to clarify the strengths and weaknesses of the four methods and to extrapolate to other realistic recording scenarios.

As we have shown, the common regression formula to calculate \( b \) gives false results if the contributions from covariance terms (covariance of true EEG and true EOG) and from forward propagation in the EOG trace are not zero. The effect of this was demonstrated in Figure 2, showing that the traces for the separately corrected up and down data were distinctly different from the AAA correction. A modification of the minimization formula for regression led to another approximating formula for \( b \), namely, by minimizing the amplitude of the corrected EEG trace. The results from the section “Experimental Data” demonstrate that this method is more stable than the regression formula. As a drawback, it should be noted that the amplitude formula is not independent of the choice of baseline. That is, its results will change if we reference the data not to the average of the data over the whole recording period.

The two algorithms presented in the section “Similarity Approaches” use different aspects of the similarity of corrected up and down traces. Our recommendation is to use either the regression (13) or the similarity of traces formula (25) for the calculation of \( b \). The first reason is the invariance to a change of baseline, which contrasts these formulas with the amplitude minimization approach. Second, the similarity of traces formula is an exact formula if the assumptions about the up-down paradigm hold, and it is still a good approximation if the assumptions are violated, as shown in the section “Similarity Approaches”. Third, compared with the similarity of means formula (20), the results seem more stable and in accordance with the minimization algorithms when reviewing the corrected traces in Figure 1.

To conclude, we have shown that one-channel EOG correction for one-dimensional eye movements can be performed in a rigorous manner and we have presented algorithms for conducting this. Using the AAA calibration paradigm to determine the backward propagation coefficient and the subtraction formula “corrected EEG = measured EEG—backward propagation \* measured EOG” to correct the EEG will provably give strong correction results.

REFERENCES


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